II Flux $F_{\bar{\nu}}^{\downarrow\uparrow}$: équ diff reliant à $M_{\bar{\nu}}^{B}$

II.1 objectif :

flux surfaciques de bande en IR tellurique

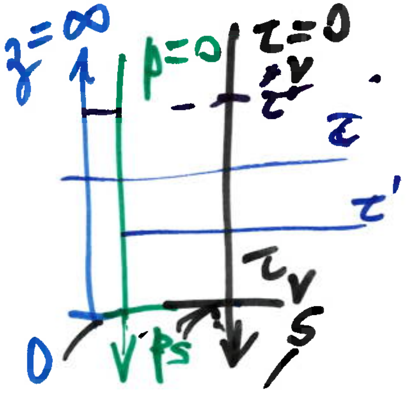
Hyp :
- IR tellurique ⇒ source thermique $J_\nu = B_\nu(T_{\tau_\nu})$
- atmosphère plan // x et y n'interviennent plus
- symétrie / angle φ
- pas diffusion $\omega_s = 0$
- absorption forte ⇒ saturation ⇒ $\Delta\tau_\nu \simeq 1$ pour interaction forte

Méthode
- solution formelle de l'équ du T.R.
- réduire nb de variables par $\int d\varphi$ $\int \mu \, d\mu$
- $\int_{\bar{\nu}-\frac{\Delta\nu}{2}}^{\bar{\nu}+\frac{\Delta\nu}{2}} d\nu$ $\Delta\nu \gg$ largeur de raies

  $\Delta\nu$ faible pour B_ν ne varie pas bcp sur $\Delta\nu$

II.2 Intégration angulaire

(12)



$\mu=\cos\theta$

$$L^{\uparrow}(\tau_\nu,\mu,\varphi) = B_\nu(T_s)\, e^{-\frac{\tau_s-\tau_\nu}{\mu}} + \int_{\tau_\nu}^{\tau_{\nu s}} B_\nu(T_{\tau'_\nu})\, e^{-\frac{\tau'_\nu-\tau_\nu}{\mu}} \frac{d\tau'_\nu}{\mu}$$

$\mu>0$ surf couches inf.

$$L^{\downarrow}(\tau_\nu,-\mu,\varphi) = 0 + \int_0^{\tau_\nu} B_\nu(T_{\tau'_\nu})\, e^{-\frac{\tau_\nu-\tau'_\nu}{\mu}} \frac{d\tau'_\nu}{\mu}$$

$\mu>0$ espace couches supér.

$$F^{\uparrow}(\tau_\nu) = \int_{\varphi=0}^{2\pi}\int_{\theta=0}^{\theta=\pi/2} L^{\uparrow}(\tau_\nu,\mu,\varphi)\cos\theta\, d^2\Omega$$

$\cos\theta \times \sin\theta\, d\theta\, d\varphi$

$-\mu\, d\mu\, d\varphi$

$\mu=\cos\theta$ $\quad d\mu = -\sin\theta\, d\theta$

$$F^{\uparrow}(\tau_\nu) = 2\pi\int_{\mu=0}^{1} L^{\uparrow}(\tau_\nu,\mu)\,\mu\, d\mu$$

$$F^{\downarrow}(\tau_\nu) = 2\pi\int_{\mu=0}^{1} L^{\uparrow}(\tau_\nu,-\mu)\,\mu\, d\mu$$

$$F^{\uparrow}(\tau_\nu) = 2\underbrace{\pi B_\nu(T_s)}_{M_\nu^B(T_s)} \int_0^1 e^{-\frac{\tau_{s} - \tau_\nu}{\mu}} \mu \, d\mu \qquad T^*(\tau_\nu)$$

$$+ 2\pi \int_{\tau_\nu}^{\tau_s} B_\nu(T_{\tau'_\nu}) \int_0^1 e^{-\frac{\tau'_\nu - \tau_\nu}{\mu}} \frac{d\tau'_\nu}{\mu} \mu \, d\mu$$

$$F^{\downarrow}(\tau_\nu) = 2\pi \int_0^{\tau_\nu} B_\nu(T_{\tau'_\nu}) \int_0^1 e^{-\frac{\tau_\nu - \tau'_\nu}{\mu}} \frac{d\tau'_\nu}{\mu} \mu \, d\mu$$

II.3 Transmissions diffuses

transmission	directionnelle	diffuse
rapport de	luminances	flux verticaux surfaciques
monochromatique	$T(\tau_\nu, \mu) = e^{-\frac{\tau_\nu}{\mu}}$	$t^*(\tau_\nu) = \frac{\int_0^1 e^{-\frac{\tau_\nu}{\mu}} \mu \, d\mu}{\int_0^1 \mu \, d\mu}$ (moy, pond.) ; $\int_0^1 \mu \, d\mu = 1/2$; moy sur directions
de bande $[\bar\nu - \frac{\Delta\nu}{2}, \bar\nu + \frac{\Delta\nu}{2}]$	$\mathcal{T}(\tau_{\bar\nu}, \mu) = \langle T(\tau_\nu, \mu) \rangle_{[\bar\nu - \frac{\Delta\nu}{2}, \bar\nu + \frac{\Delta\nu}{2}]}$	$\mathcal{T}^*(\tau_{\bar\nu})$

Exponentielle intégrale E_n

$$E_n(\tau) = \int_1^{\infty} \frac{e^{-\tau x}}{x^n}\, dx \qquad \mu = \frac{1}{x} \quad d\mu = -\frac{dx}{x^2} \qquad dx = -\frac{d\mu}{\mu^2}$$

$$E_n(\tau) = \int_0^1 e^{-\frac{\tau}{\mu}} \mu^{n-2}\, d\mu$$

$$\frac{dE_n}{d\tau} = -\cancel{\frac{1}{\mu}} \int_0^1 e^{-\frac{\tau}{\mu}} \mu^{n-3}\, d\mu = -E_{n-1}(\tau)$$

$$T^*(\tau) = 2\int_0^1 e^{-\frac{\tau}{\mu}} \mu\, d\mu = 2\, E_3(\tau)$$

$$\frac{\partial T^*}{\partial \tau}(\tau) = -2\int_0^1 \frac{e^{-\frac{\tau}{\mu}}}{\cancel{\mu}} \cancel{\mu}\, d\mu = -2\, E_2(\tau) < 0$$

Expression de $F^{\uparrow\downarrow}$ en fct de T^* et $\frac{\partial T^*}{\partial \tau}$

$$F^{\uparrow}(\tau_\nu) = M_\nu^B(T_{surf})\, T^*(\tau_{surf} - \tau_\nu) - \int_{\tau_\nu}^{\tau_S} M_\nu^B(T_{\tau'_\nu}) \frac{\partial T^*}{\partial \tau'_\nu}(\tau'_\nu - \tau_\nu)\, d\tau'_\nu$$

$$F^{\downarrow}(\tau_\nu) = \int_0^{\tau_\nu} M_\nu^B(T_{\tau'_\nu}) \frac{\partial T^*}{\partial \tau'_\nu}(\tau_\nu - \tau'_\nu)\, d\tau'_\nu$$

II.4 Approx du noyau exponentielle

ν fixe $\quad T^*(\tau_\nu) = \dfrac{\int_0^1 e^{-\tau_\nu/\mu} \mu\, d\mu}{\int_0^1 \mu\, d\mu} = 2E_3(\tau_\nu) = e^{-\frac{\tau_\nu}{\bar{\mu}}} = T(\tau_\nu, \bar{\mu})$

$\bar{\mu}_\nu$ = fct de τ_ν, donc de ν

Expression des flux en fct de l'épaisseur optique oblique à $\bar{\mu}$ (16)

$\tau^*_{\bar{\nu}} = \frac{\tau_{\bar{\nu}}}{\bar{\mu}}$ oblique

$$F^{\uparrow}_{\bar{\nu}}(\tau^*_{\bar{\nu}}) = M^{B}_{\bar{\nu}}(T_S)\, e^{-(\tau^*_{S}-\tau^*_{\bar{\nu}})} + \int^{\tau_{\bar{\nu}\,surf}} M_{\bar{\nu}}(\tau'^*_{\bar{\nu}})\, e^{-(\tau'^*_{\bar{\nu}}-\tau^*_{\bar{\nu}})}\, d\tau'^*_{\bar{\nu}}$$

$$F^{\downarrow}_{\bar{\nu}}(\tau^*_{\bar{\nu}}) = \int_0^{\tau^*_{\bar{\nu}}} M^{B}_{\bar{\nu}}(\tau'^*_{\bar{\nu}})\, e^{-(\tau^*_{\bar{\nu}}-\tau'^*_{\bar{\nu}})}\, d\tau'^*_{\bar{\nu}}$$

$$F^{\uparrow}(\tau) = M(T_S)\, e^{-(\tau_S-\tau)} + \int^{\tau_S} M(\tau')\, e^{-(\tau'-\tau)}\, d\tau'$$

$$F^{\downarrow}(\tau) = \int_0^{\tau} M(\tau')\, e^{-(\tau-\tau')}\, d\tau'$$

$$\frac{dF^{\uparrow}}{d\tau}$$

$$\frac{dF^{\downarrow}}{d\tau}$$

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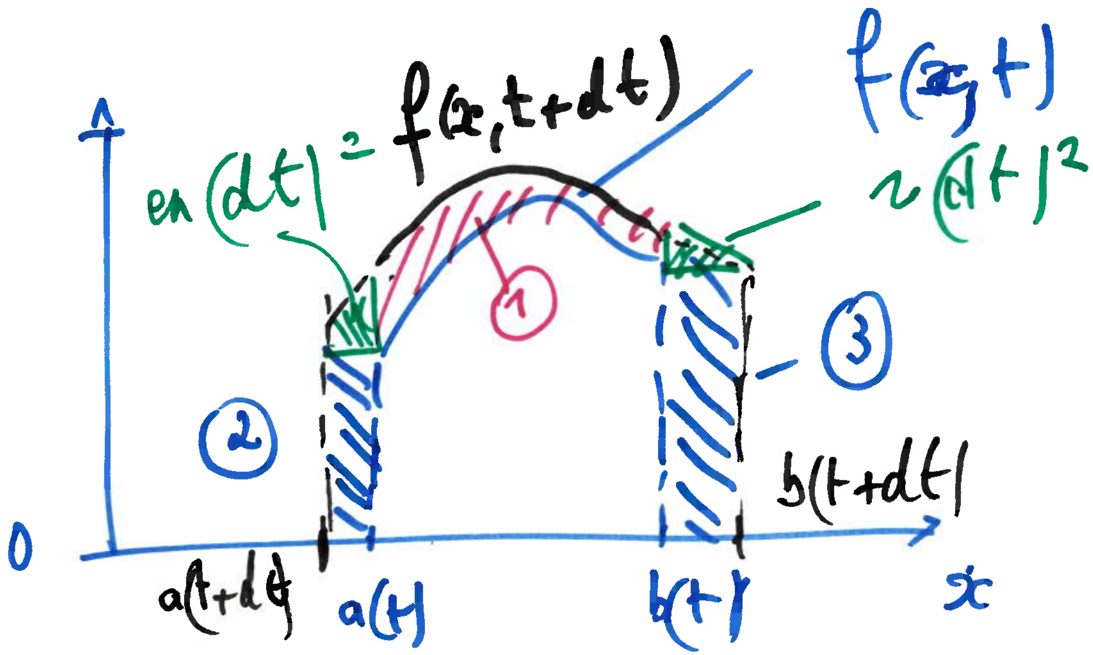


$$I(t) = \int_{a(t)}^{b(t)} f(x,t)\,dx$$

Q: $\dfrac{dI}{dt} = ?$

$$\frac{I(t+dt) - I(t)}{dt} \qquad I(t+dt) - I(t) = \int_{a(t)}^{b(t)} \Big[\underbrace{f(x,t+dt) - f(x,t)}_{\frac{\partial f}{\partial t}\cancel{dt}}\Big]\,dx \quad 1$$

$$+ \int_{a(t+dt)}^{a(t)} f(x,t)\,dx \qquad 2 \simeq -\frac{\partial a}{\partial t}\cancel{dt}\, f(a,t)$$

$$+ \int_{b(t)}^{b(t+dt)} f(x,t)\,dx \qquad 3 \simeq \frac{\partial b}{\partial t}\cancel{dt}\, f(b,t)$$

$$\frac{\partial I}{\partial t} = \int_{a(t)}^{b(t)} \frac{\partial f}{\partial t}\,dx + \frac{\partial b}{\partial t} f(b,t) - \frac{\partial a}{\partial t} f(a,t)$$

$$\frac{dF^{\uparrow}}{d\tau} = \underbrace{M(\tau_s)\, e^{-(\tau_s-\tau)} + \int_{\tau}^{\tau_s} M(\tau')\, e^{-(\tau'-\tau)}\, d\tau'}_{F^{\uparrow}} - M(\tau)$$

$$\frac{dF^{\downarrow}}{d\tau} = M(\tau) - \int_0^{\tau} M(\tau')\, e^{-(\tau-\tau')}\, d\tau = M - F^{\downarrow}$$

$$\boxed{\begin{aligned} \frac{dF^{\uparrow}}{d\tau} &= F^{\uparrow} - M \\ \frac{dF^{\downarrow}}{d\tau} &= -F^{\downarrow} + M \end{aligned}}$$

III Méthodes à deux flux

III.1 Introd

équilibre radiatif troposphère

Hyp :

- atmosphère transparente au solaire sauf albedo (pas d'absorption)
- énergétique $\Rightarrow \int d\nu$:

atmosphère sauf diff et prise en IR tellurique

τ_ν indépendant de ν $F^\uparrow(\tau) = \int_\nu F^\uparrow(\tau_\nu)\, d\nu$

C. Limites

- $\Phi_0 = \frac{(1-A)C}{4}$ absorbé en surface

  $F^\uparrow(\tau=0)$ doit équilibrer Φ_0

- surface = C.N.

Supposant équilibre radiatif $F_{net} = F^\uparrow - F^\downarrow$ indépt de z

$\frac{\partial F}{\partial \tau} = 0$

III.2 Résolution

$F = F^{\uparrow} - F^{\downarrow}$ $\quad \bar{F} = F^{\uparrow} + F^{\downarrow}$

$$\left| \begin{aligned} \frac{dF^{\uparrow}}{d\tau} &= F^{\uparrow} - M \\ \frac{dF^{\downarrow}}{d\tau} &= -F^{\downarrow} + M \end{aligned} \right| \quad \begin{aligned} &\ominus \\ &\oplus \end{aligned} \quad \left| \begin{aligned} \frac{dF}{d\tau} &= \bar{F} - 2M = 0 \\ \frac{d\bar{F}}{d\tau} &= F = F(0) = \Phi_0 \end{aligned} \right.$$

$$\bar{F} = \bar{F}(0) + \Phi_0 \tau$$

$$\bar{F}(0) = F^{\downarrow}(0) + F^{\uparrow}(0) = 0 + \Phi_0$$

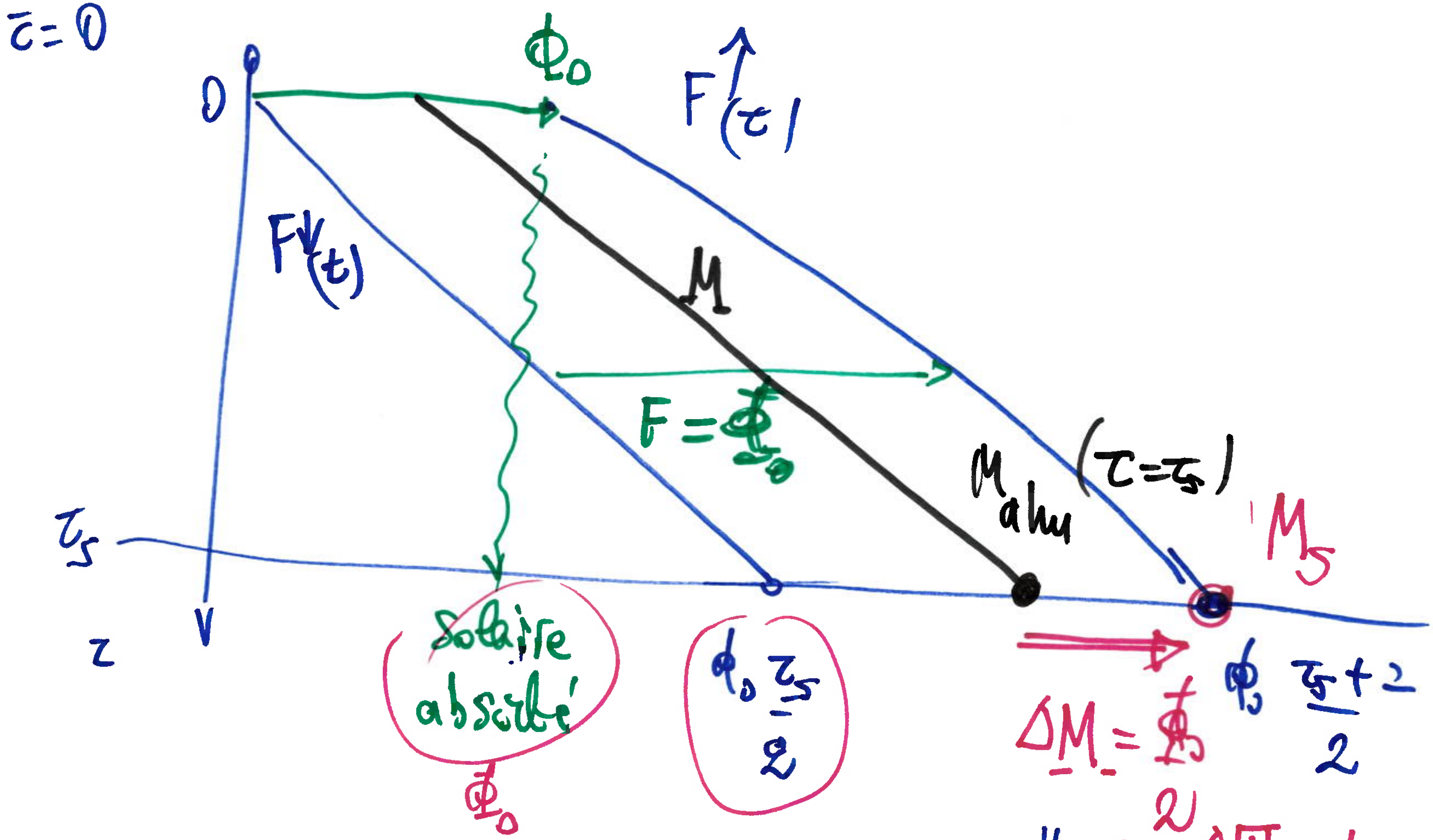
$$\left\{ \begin{aligned} \bar{F} &= \Phi_0 (1+\tau) \\ F &= \Phi_0 \end{aligned} \right.$$

$$\left| \begin{aligned} F^{\uparrow} &= \frac{\bar{F}+F}{2} \\ F^{\downarrow} &= \frac{\bar{F}-F}{2} \end{aligned} \right.$$

$$\boxed{\begin{aligned} F^{\uparrow} &= \Phi_0 \frac{\tau+2}{2} \\ F^{\downarrow} &= \Phi_0 \frac{\tau}{2} \end{aligned}}$$

$$M = \Phi_0 \frac{\tau+1}{2}$$

Solutions



$\Rightarrow \Delta T$ entre surf et atmos

$\Downarrow$

équilibre radiatif impossible