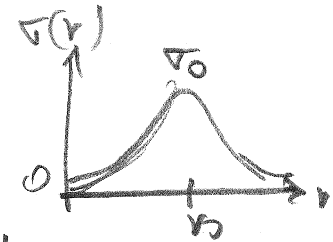
TD 5/E4

Répartition spectrale de l'absorption

IR tellurique ↑ | sans émission ! | sans diffusion



absorbant X_i $\quad n_i(z)$ $\quad \sigma(\nu)$ $\forall z$

σ max (σ_0) à ν_0 $\quad \sigma(\nu) \searrow$ si $|\nu-\nu_0| \nearrow$

$\sigma(\nu) \to 0$ si $|\nu-\nu_0|$ gd

$\phi(\nu, z=0) = \phi_0$

1) $\dfrac{d\phi}{dz} = -\sigma(\nu)\, n_i(z)\, \phi(\nu, z)$

2) $\tau(\nu, z) = \sigma(\nu) \int_0^z n_i(z')\,dz' = \sigma(\nu)\, N_i(z)$

$n_i(z) = n_i(0)\exp(-z/H_i)$

$N_i(z) = n_i(0)\int_0^z e^{-z'/H_i}dz' = n_i(0)\,H_i\left[1-e^{-z/H_i}\right]$

$\boxed{N_i(z) = H_i\left[n_i(0) - n_i(z)\right]}$ $\quad N_i(\infty) = H_i\, n_i(0)$

$\tau(\nu,z) = \sigma(\nu)\,H_i\,n_i(0)\left[1-e^{-z/H_i}\right]$

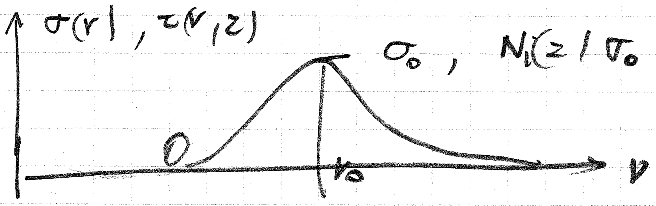
3) $T(\nu,z) = \dfrac{\phi(\nu,z)}{\phi_0}$ $\quad \dfrac{1}{\phi}\dfrac{d\phi}{dz} = -\sigma(\nu)\,n_i(z)$

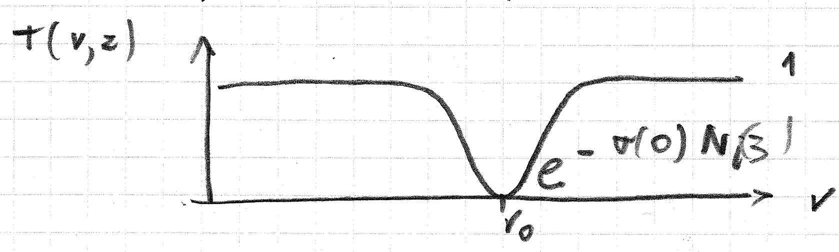
$\dfrac{d\ln\phi}{dz} = -\sigma(\nu)\,n_i(z) = -\dfrac{d\tau_i}{dz}$

$\ln\dfrac{\phi}{\phi_0} = -\tau_i(\nu,z) \Rightarrow T(\nu,z) = e^{-\tau_i(\nu,z)}$

$T(\nu,z) = \exp\left[-\sigma(\nu)\,N_i(z)\right]$

z fixé





$R(\nu,z) = -\dfrac{d\phi}{dz}(\nu,z)$ tx de transfert rayt → milieu

4) $R = -\dfrac{d\phi}{dz} = \sigma(\nu)\, n_i(z)\, \phi(\nu,z) = \dfrac{d\tau}{dz}\, \phi(\nu,z)$

$$R(\nu,z) = \phi_0\, e^{-\tau(\nu,z)}\, \sigma(\nu)\, n_i(z)$$

5) z fixé $\quad \dfrac{1}{R}\dfrac{dR}{d\nu} = \dfrac{1}{\sigma}\dfrac{d\sigma}{d\nu} - \dfrac{d\tau}{d\nu}$

$$= \left(\frac{1}{\sigma(\nu)} - N_i(z)\right)\frac{d\sigma}{d\nu}$$

$$\boxed{\frac{1}{R}\frac{dR}{d\nu} = (1-\tau(\nu,z))\,\frac{1}{\sigma(\nu)}\,\frac{d\sigma}{d\nu} = 0}$$

$\Leftrightarrow \begin{cases} \underline{\tau(\nu,z) = 1} \\ \text{ou} \\ \dfrac{d\sigma}{d\nu} = 0 \quad \Leftrightarrow \quad \underline{\nu = \nu_0} \end{cases}$

6) a) $\tau(\nu_0,z) < 1$ abs. faible

$\forall \nu \quad \tau(\nu,z) < 1$

ν		ν_0	
$\dfrac{dR}{d\nu}$	+	0	−
R	↗		↘

$\tau^* = \tau(\nu_0,z) = \sigma_0 N_i(z)$

$R_{max} = \phi_0\, e^{-\sigma_0 N_i(z)}\, \sigma_0\, n_i(z)$

b) $\tau(\nu_0,z) > 1$ abs. forte

ν		ν_-		ν_0		ν_+	
$1-\tau$	+	0	−		−	0	+
$\dfrac{d\sigma}{d\nu}$	+		+	0	−		−
$\dfrac{dR}{d\nu}$	+		−		+		−
R	↗		↘		↗		↘

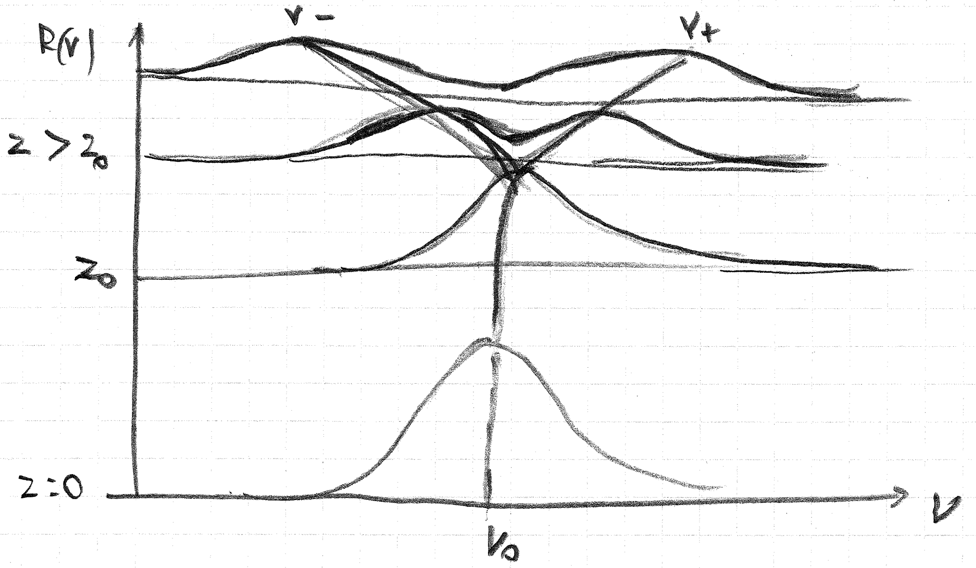
R_{max} pour $\tau^* = 1$ $\quad R_{max} = \frac{\phi_0}{e}\, n_i(z)\, \sigma(\nu^*)$

$$R_{max} = \frac{\phi_0}{e} \frac{n_i(z)}{N_i(z)} \tau^* = \frac{\phi_e}{e} \frac{n_i(z)}{N_i(z)}$$

si $n_i(z) = n_i(0)\, e^{-z/H_i}$

$$R_{max} = \frac{\phi_e}{e H_i} \frac{e^{-z/H_i}}{1 - e^{-z/H_i}} = \frac{\phi_e}{eH} \frac{1}{e^{z/H_i} - 1}$$

7) $\sigma N_i(\infty) = \tau(\nu_0, \infty) > 1$ $\quad \exists\, z_0$ t.q $\sigma N_i(z_0) = 1$



$z > z_0$ 2 max $|^{\nu_-}_{\nu_+}$

$z < z_0$ 1 max à ν_0